Conditional Probability: the probability of an event changes because of information given

Ex.

<table>
<thead>
<tr>
<th></th>
<th>Daily exercise</th>
<th>Occasional</th>
<th>Never</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>157</td>
<td>304</td>
<td>82</td>
<td>543</td>
</tr>
<tr>
<td>Female</td>
<td>123</td>
<td>328</td>
<td>77</td>
<td>528</td>
</tr>
<tr>
<td>Total</td>
<td>280</td>
<td>632</td>
<td>159</td>
<td>1071</td>
</tr>
</tbody>
</table>

What is the probability that we select a) a female, b) a male that never exercises c) a daily exerciser?

\[
\frac{280}{1071} \quad \frac{528}{1071} \quad \frac{82}{1071}
\]

If we know that a selected student is a female, does that effect the probability that they are a daily exerciser? What is that probability?

\[
P(D \text{ given they are female}) = \frac{123}{528}
\]

Multiplication Rule: Given any events A and B, the probability that both events happen is given by

\[
P(A \cap B) = P(A \mid B) \times P(B)
\]

Or when \(P(B) \neq 0\), the conditional probability of A given B is:

\[
P(A \mid B) = \frac{P(A \cap B)}{P(B)}
\]

Two events are independent if and only if either \(P(A \cap B) = P(A) \times P(B)\), or \(P(A \mid B) = P(A)\). Otherwise, the events are dependent.

Example 1: The probability that a Air Canada flight will depart on time is, \(P(D) = 0.8\)
The probability that it arrives on time is \( P(A) = 0.91 \) and the probability that it departs and arrives on time, \( P(D \cap A) = 0.76 \).

Find the probability that a flight:

a) Arrives on time given that it departed on time

\[
P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{0.76}{0.8} = 0.95
\]

b) Departs on time given it arrived on time

\[
P(D|A) = \frac{P(D \cap A)}{P(A)} = \frac{0.76}{0.91} = 0.835
\]

Example 2: Assume 81% of drivers stopped at a road check are given a breath test, 20% a blood test, and 15% both.

a) What is the probability that a suspended driver is given:
   a. a test?
      \[
      0.86
      \]
   b. Exactly one test?
      \[
      0.66 + 0.05 = 0.71
      \]
   c. No test?
      \[
      0.14
      \]

b) Are giving the two tests independent?

\[
\text{if and only if } P(Br \cap B1) = P(Br) \times P(B1)
\]

\[
0.15 = 0.81 \times 0.20
\]

\[
0.15 \neq 0.162 \times
\]

\[\text{Not independent}\]