We use the sin law to determine the missing angle or side of triangles that are not right.

Two problems we need to solve:

1. Solve for a missing side
2. Solve for a missing angle

**Equation:**

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

1. Solve for a missing side

Example: Determine \( x \)

\[
\frac{12}{\sin 40^\circ} = \frac{x}{\sin 60^\circ}
\]

\[
\frac{12}{0.643} = \frac{x}{0.866}
\]

\[
x = 12 \times \left( \frac{0.866}{0.643} \right) = 16.2
\]

Try on your own: solve for \( x \)
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(2) Solve for a missing angle

Example: determine $\theta$

\[
\frac{25}{\sin 100^\circ} = \frac{18}{\sin \theta}
\]

\[
\sin \theta = 18 \times (\sin 100^\circ) \div 25
\]

\[
\sin \theta = 0.70906
\]

\[
\sin^{-1}(0.70906) = 45.2^\circ
\]

Try on your own: determine $\theta$

\[
\frac{17}{\sin 96^\circ} = \frac{13}{\sin \theta}
\]

\[
\sin \theta = 13 \times (\sin 96^\circ) \div 17
\]

\[
\sin \theta = 0.7605
\]

\[
\theta = 49.5^\circ
\]
The ambiguous case of sine law

\[ \frac{5}{\sin 30^\circ} = \frac{7}{\sin x} \]

\[ \sin x = \frac{7 \times \sin 30^\circ}{5} \]

\[ \sin x = 0.7 \]

\[ x = 44.5^\circ \]

\[ \Theta = 180 - 30 - 44.5 \]

\[ \Theta = 105.5^\circ \]

Examples: Solve for \( x \) and \( y \)

a) \[
\begin{array}{c}
12m \\
37^\circ \\
90^\circ \\
x
\end{array}
\]

b) \[
\begin{array}{c}
126^\circ \\
82 km \\
105 km \\
x
\end{array}
\]

\( \Theta = 105^\circ \)

Your reference angle is 75°, so our two triangles look like

\[
\begin{array}{c}
7cm \\
30^\circ \\
5cm \\
\end{array}
\]