6.3 Translating Sine & Cosine Graphs \( y = a \sin(x + c) + d \)

- **a** – amplitude (Chapter 2 term – vertical expansion/compression)
- **c** – phase shift (Chapter 2 term – horizontal translation)
- **d** – vertical displacement (Chapter 2 – vertical translation)

**Graphing:**

- **Vertical expansion x 2**
  - **Graph:**
  - **Range:** \(-2 \leq y \leq 2\)
  - **Maximum:** 2
  - **Minimum:** -2
  - **Amplitude:** 2

- **Reflection over x-axis stretching by a factor of 3**
  - **Graph:**
  - **Range:** \(-3 \leq y \leq 3\)
  - **Maximum:** 3
  - **Minimum:** -3
  - **Amplitude:** 3

- **Vertical displacement:**
  - **Graph:**
  - **Range:** \(1 \leq y \leq 3\)
  - **Maximum:** 3
  - **Minimum:** 1
  - **Vertical Displacement:** **up 2**

- **Graph:**
  - **Range:** \(-2 \leq y \leq 0\)
  - **Maximum:** 0
  - **Minimum:** -2
  - **Vertical Displacement:** **down 1**

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Graphing Tips (for chapter 6 only)
1. Do vertical displacement and amplitude at the same time! (only because of the periodic nature of the functions)
2. Then apply the phase shift (translate left/right)

Formulae:

\[
\text{amp} = \frac{\text{MAX} - \text{MIN}}{2} \quad \text{VD} = \frac{\text{MAX} + \text{MIN}}{2} \quad \text{MAX} = d + |a| \quad \text{MIN} = d - |a|
\]
Graph: \( y = 3 \sin \left( x + \frac{\pi}{3} \right) + 1 \)

- \( \text{move left} \ \frac{\pi}{3} \) (2 lines)
- \( \text{move up} \ 1 \)

<table>
<thead>
<tr>
<th>Amplitude: 3</th>
<th>Vertical Displacement: up 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum(s) 4</td>
<td>Minimum(s): -2</td>
</tr>
<tr>
<td>Phase Shift: ( \frac{\pi}{3} ) left +</td>
<td>Period: I know... boring 2\pi</td>
</tr>
</tbody>
</table>

Write the function graphed below in as many ways as you can.
6.3 Graphing \( y = a \sin(b(x - c)) + d \) and \( y = a \cos(b(x - c)) + d \)

- **a** – Amplitude
- **b** – Period change (chapter 2 – horizontal expansion/compression)
- **c** – Phase shift
- **d** – Vertical displacement

In chapter 2, we graphed vertical expansion/compressions, next, horizontal expansion/compressions, and then translation left/right, and finally up/down.

When graphing in chapter 6 we can graph in a slightly different order to speed up the process. We can do this because of the repetitive(periodic) function as well as the range of the initial function is always \(-1 \leq y \leq 1\). The order for chapter 6:

1. Graph vertical displacement & amplitude at the same time
2. Graph the period change
3. Phase shift the graph

**For example:**

Graph: \( y = 3 \sin \left( 2 \left( x - \frac{\pi}{6} \right) + 1 \right) \)

**Amplitude:** 3
**Vertical Displacement:** 1
**Phase Shift:** \( \text{right} \ \frac{\pi}{6} \)
**Period:** \( \pi \)
**Maximum:** 4
**Minimum:** -2

NOTE: The scale of the graph is important. Count the number of “squares” from 0 to 2\( \pi \) to determine the scale of the graph.
Determine the following properties of the function without graphing:

\[ y = 7 \cos \left( \frac{2}{3} \left( x - \frac{\pi}{6} \right) \right) - 4 \]

Amplitude:
Vertical Displacement:
Phase Shift:
Period:
Maximum:
Minimum:

\[ y = -7 \sin \left( \frac{9}{2} \left( x - \frac{\pi}{2} \right) \right) + 2 \]

Amplitude:
Vertical Displacement:
Phase Shift:
Period:
Maximum:
Minimum:

Fast way to calculate the period: \( period = \frac{2\pi}{b} \) ... but we can also use the ideas from chapter 1 (thinking it as a horizontal compression/expansion).
Example: A sinusoidal function has a maximum at \( \left( \frac{\pi}{8}, 12 \right) \), the next minimum is at \( \left( \frac{3\pi}{4}, -4 \right) \). Determine a sinusoidal function that best represents this situation (hint use cosine...)

\[ y \]

\[ x \]
6.3 Graphing \( y = a \sin \frac{2\pi}{p}(x - c) + d \) and \( y = a \cos \frac{2\pi}{p}(x - c) + d \)

When graphing \( y = \sin \frac{2\pi}{b}x \) or \( y = \cos \frac{2\pi}{b}x \), the period is \( \frac{2\pi}{b} \). For example, the period of \( y = \sin (3x) \) is \( \frac{2\pi}{3} \). However, when the function is of the form \( y = \sin \frac{2\pi}{p}x \), the period is \( p \).

For example, find the period of \( y = \sin \frac{2\pi}{10}x \).

\[
\text{Period} = \frac{2\pi}{\frac{2\pi}{10}} = 5 \Rightarrow \frac{2\pi}{\frac{2\pi}{10}} = 2 \times \frac{2\pi}{10} = 10
\]

Do it quickly to find the period:

<table>
<thead>
<tr>
<th>( y = \sin \frac{2\pi}{9}x )</th>
<th>( y = \cos \frac{2\pi}{7}x )</th>
<th>( y = \sin \frac{\pi}{6}x )</th>
<th>( y = \sin (\pi x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period = 9</td>
<td>Period = 7</td>
<td>Period = 12</td>
<td>Period = 2</td>
</tr>
</tbody>
</table>

Graph:

\[
y = \sin \left( \frac{2\pi}{10}x \right) \quad \text{Period} = 10
\]

\[
y = \cos \left( \frac{2\pi}{8}x \right) \quad \text{Period} = 8
\]

\[
y = 3 \cos \left( \frac{2\pi}{12}x \right) + 2 \quad \text{Period} = 12
\]
A sinusoidal (sine or cosine function) has a first maximum at (2, 10) and has a first minimum at (4, -2).

\[ y = a \sin(b(x+c)) + d \]

- \( a = 6 \)
- \( b = 2\pi \frac{12}{\pi} = \frac{\pi}{2} \)
- \( c = -1 \) shift right + 1
- \( d = 4 \) up + 4

\[ y = 6 \sin\left(\frac{\pi}{6}(x-1)\right) + 4 \]