an angle is measured in the units degrees 
> there are 360° in a complete circle

- a degree is \( \frac{1}{360} \) of a circle

- draw a 90° angle and a 45° angle

Angles

- when the initial side will lie on the positive x-axis
- if the terminal arm rotates counter clockwise, we call this a positive angle
- if the terminal arm rotates clockwise we call it a negative angle

Examples

\[ \theta = 120° \]

\[ \theta = -240° \]
- Two angles in standard position that have the same terminal side are called **coterminial**.

- We can add or subtract multiples of 360° to get coterminial angles to an angle in standard position.

**Example:**

If the angle is 170°, then

\[ 170° + 360° = 530° \] is coterminial.

**Radian:** One radian is the angle subtended at the center of a circle by an arc of length equal to the radius of the circle.

This angle is 1 radian.
Converting between Degrees and Radians

1 radian = \( \frac{180^\circ}{\pi} \approx 57.3^\circ \)

Convert radians to degrees:

a) \( 4 \text{ radians} \approx 4 \times 57.3^\circ \approx 229.2^\circ \)  
   \( \approx \) approximation

   \( 4 \text{ radians} = 4 \times \frac{180^\circ}{\pi} = \frac{720^\circ}{\pi} \)  
   \( \approx \) exact answer

b) \( \frac{3\pi}{2} \text{ radians} = \frac{3\pi}{2} \times \frac{180^\circ}{\pi} = 270^\circ \)

Convert from degrees to radians:

a) \( 120^\circ = 120^\circ \times \frac{\pi}{180^\circ} = \frac{120\pi}{180} = \frac{2\pi}{3} \text{ radians} \)

b) \( 315^\circ = 315^\circ \times \frac{\pi}{180^\circ} = \frac{315\pi}{180} = \frac{7\pi}{4} \text{ radians} \)

Quick conversion:

radian \( \times \frac{180^\circ}{\pi} \rightarrow \) degree

degree \( \times \frac{\pi}{180^\circ} \rightarrow \) radian
For a circle of radius $r$, a central angle $\theta$ subtends an arc of the circle of length $l$ given by:

$$l = r \theta$$

where $\theta$ is in radian measure.

**Example:** Determine the length of an arc with an angle of $\frac{\pi}{2}$ radians and radius of 10 cm.

**Solution**

$$\frac{\pi}{2} \text{ radians} \times \frac{180^\circ}{\pi} = 90^\circ$$

$$l = r \theta$$

$$l = (10 \text{ cm}) \left( \frac{\pi}{2} \right)$$
\[ l = (10\text{ cm})(\frac{\pi}{2}) \]
\[ l = \frac{10\pi}{2} = 5\pi \approx 15.71\text{ cm} \]

Determine the radius of a circle, if the arc length is 27 cm and we have an angle of 125°.

**Solution**

\[ \theta = 125° \times \frac{\pi}{180°} = \frac{25\pi}{36} \]

\[ l = r\theta \]

\[ (\frac{36}{25\pi}) \times 27 = r \times \left(\frac{25\pi}{36}\right) \times \left(\frac{36}{25\pi}\right) \]

\[ 12.38\text{ cm} = r \]

**Area of a sector**

A sector is a region bounded by an arc and two sides of a central angle.
two sides of a central angle

area of a sector:

\[ A = \frac{1}{2} \theta r^2 \quad \theta = \text{radian measure} \]
\[ r = \text{radius} \]

Example: In a circle of radius 9 cm with a central angle of \( \frac{2\pi}{3} \) radians. Determine the area of the sector.

\[ \text{solution} \]

\[ A = \frac{1}{2} \theta r^2 \]
\[ A = \frac{1}{2} \left( \frac{2\pi}{3} \right) (9)^2 \]
\[ A = \frac{2\pi \times 81}{6} = 27\pi \]
\[ A = 84.8 \text{ cm}^2 \]

p. 171 # 25 - 33 odd
# 5

- read 6.2