We use logs to help solve equations that have a variable as an exponent.

**Example:**

(i) \( 2^x = 182 \)

Convert to a logarithm:

\[
\log_2 182 = x
\]

\[
\log 182 \quad \frac{x}{\log 2}
\]

\[7.508 = x\]

**Example:** A population of 200 bacteria doubles every day. How long will it take to reach a population of 13,000?

**Solution:**

\[
P = P_0 (r)^{\frac{t}{n}}
\]

\[
13000 = 200(2)^{\frac{t}{1}}
\]

\[
\frac{13000}{200} = 200
\]

\[
P_0 = \text{initial population}\]

\[r = \text{common ratio}\]

\[t = \text{time}\]

\[n = \text{# of times double in a day}\]
\[
65 = 2^t \\
\Rightarrow \log_2 65 = t \\
\frac{\log 65}{\log 2} = t \\
60.2 = t
\]

\[
\Rightarrow 65 = 2^t \\
\log 65 = \log 2^t \\
\frac{\log 65}{\log 2} = \frac{\log 2^t}{\log 2} \\
\log 65 = t \log 2 \\
\frac{\log 65}{\log 2} = t \\
60.2 = t
\]

**Example:**

\[
3^{x+2} = 24 \\
\log 3^{x+2} = \log 24 \\
(\log 3 \cdot x + \log 3) = \log 24 \\
\]
\[
(x + 2) \log 3 = \log 24 \quad \Rightarrow \quad \text{divide both sides by } \log 3 \\
x + 2 = \frac{\log 24}{\log 3} \quad \Rightarrow \quad \text{subtract 2 from both sides}
\]

\[
x = \frac{\log 24}{\log 3} - 2
\]

\[
\boxed{x = 0.8928}
\]

Read section 4.5
p. 134-140

Example: The population of Canada is about 34.2 million. It grows at a rate of 0.9% annually. How long will it take to double in population?

Solution

Final population = 68.4 million
Initial population = 34.2 million
Growth rate = 0.9% = 0.009
\[ P = P_0 \left( r \right)^t \]

\[
\frac{68.4}{34.2} = \frac{34.2 \left( 1.009 \right)^t}{34.2} - \text{divide by 34.2}
\]

\[ 2 = \left( 1.009 \right)^t - \log \text{ both sides} \]

\[ \log 2 = \log \left( 1.009 \right)^t - \text{use power law} \]

\[ \log 2 = t \log \left( 1.009 \right) - \text{divide by} \log \left( 1.009 \right) \]

\[ \frac{\log 2}{\log \left( 1.009 \right)} = t \]

\[ 77.36 = t \]

**Examples:**

(i) \[ 2^{x+2} = 3^x \]

**Solution**

- \[ \log \text{ both sides} \]
- \[ \log 2^{x+2} = \log 3^x \]
- \[ \log \left( \times \right) \log 2 - x \log 3 \]
\((x+2) \log 2 = x \log 3\)

\[x \log 2 + 2 \log 2 = x \log 3\]

\[x \log 2 = x \log 3 - 2 \log 2\]

\[x \log 2 - x \log 3 = -2 \log 2\]

\[x (\log 2 - \log 3) = -2 \log 2\]

\[x = \frac{-2 \log 2}{\log 2 - \log 3}\]

\[x = 3.419022\ldots\]

**P. 140 # 1, 3, 5, 9, 15**

**Solving logarithmic equations.**

**Example:** solve for \(x\)

\[\log_3 (2x-5) = 2\]

Convert to exponential form:

\[3^2 = 2x - 5\]

\[9 = 2x - 5\]
14 = 2x

\[ 7 = x \]

**Solve for x:**

\[ \log_2 x + \log_2 (10-x) = 4 \]

**Use log laws to convert to a single log**

\[ \log_2 (x(10-x)) = 4 \]

**Convert to an exponent**

\[ 2^4 = x(10-x) \]

\[ 16 = 10x - x^2 \]

\[ x^2 - 10x + 16 = 0 \]

\[ (x-8)(x-2) = 0 \]

\[ x = 8 \text{ or } 2 \]

**Solve for x:**

\[ \ln (x-2) + \ln(2x-3) = 2 \ln x \]

**"Un-log" both sides**

\[ \ln (x-2)(2x-3) = \ln x^2 \]

\[ \sqrt{x-2} = \sqrt{2x-3} \]

\[ \sqrt{x-2} = \sqrt{2x-3} \]
\[(x-2)(2x-3) = x^2 \quad \text{- expand}\]
\[2x^2 - 3x - 4x + 6 = x^2\]
\[2x^2 - 7x + 6 = x^2\]
\[-x^2 - x^2\]
\[x^2 - 7x + 6 = 0 \quad \text{- factor}\]
\[(x-6)(x-1) = 0\]

\[x = 6 \quad \text{or} \quad 1\]

Original \[\ln(x-2) + \ln(2x-3) = 2\ln x\]

* We can only log or ln positive values *

If \(x = 1\) we get \[\ln(1-2) + \ln(2\cdot 1-3) = 2\ln(1)\]
\[\ln(-1) + \ln(-1) = 2\ln(1)\]

\(x = 1\) is an extraneous solution (it is not valid)

p. 141 # 23, 28, 29