Series is a sum of the terms in a sequence

\[
\text{Arithmetic} \\
S_n = \frac{n}{2} (2a_1 + (n-1)d)
\]

the sum of the first \( n \) terms

\[S_n = \frac{n}{2} (a_1 + a_n)\]

\[\text{Examples}\]

Determine the sum of the first 20 terms of the sequence 6, 11, 16, 21, ....

**Solution**

\[n = 20\]
\[a_1 = 6\]
\[d = 5\]

\[S_{20} = \frac{20}{2} (2(6) + (20-1)5)\]
\[= 10(12 + 95)\]
\[= 10(107)\]
\[S_{20} = 1070\]

**Geometric Series**

\[S_n = \frac{a_1(1-r^n)}{1-r} = \frac{a_1(1-r^n)}{1-r}, \quad r \neq 1\]
Example: Find the sum after 30 days if on day 1 you get $0.01, day 2 $0.02, day 3 $0.04, day 4 $0.08...

Solution

\[ u_1 = 0.01 \]
\[ n = 30 \]
\[ r = 2 \]

\[ S_n = \frac{u_1( r^n - 1 )}{ r - 1 } \]
\[ S_{30} = 0.01 \left( 2^{30} - 1 \right) \]
\[ S_{30} = \$10,737,418.23 \]

Infinite Geometric Series

Zeno's Paradox

Think is a wall 16 m away from you. You walk half the distance to the wall and stop. Then you walk half the distance again and stop. You continue with this pattern. You will never reach the wall.

Solution

Geometric sequence: \( 8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \cdots \)

\[ S_\infty = \frac{u_1}{1 - r}, \quad |r| < 1 \]
\[ \mu_1 = 8 \\
\gamma = \frac{1}{2} \]

\[ S_\infty = \frac{8}{1 - \frac{1}{2}} = \frac{8}{\frac{1}{2}} = 16 \]

**Sigma Notation**

Sigma (\( \Sigma \)) is a symbol to represent repeated addition.

**Examples**

\[ \sum_{i=1}^{6} i \]

- When \( i = 1 \), substitute 1 into the function.
  \[ \Rightarrow \text{we get a value of 1} \]
- When \( i = 2 \), substitute 2 into the function.
  \[ \Rightarrow \text{we get a value of 2} \]
- \( \cdots \)
- When \( i = 6 \), substitute 6 into the function.
  \[ \Rightarrow \text{we get a value of 6} \]
Arithmétique series = 1 + 2 + 3 + 4 + 5 + 6
= 21

Example:
\[ \sum_{k=7}^{12} 2(k+3) = \]
\[
2(7+3) = 20 \\
2(8+3) = 22 \\
2(9+3) = 24 \\
2(10+3) = 26 \\
2(11+3) = 28 \\
2(12+3) = 30 \\
\]
\[ 150 \]

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