Zero/root: is where the parabola crosses the x-axis, where \( y = 0 \).

**Examples:**

\[
y = (x-3)^2 - 4
\]

Substitute \( x = 1 \) and \( x = 5 \) into the equation:

When \( x = 1 \):
\[
\begin{align*}
y &= (1-3)^2 - 4 \\
y &= (-2)^2 - 4 \\
y &= 4 - 4 \\
y &= 0
\end{align*}
\]

When \( x = 5 \):
\[
\begin{align*}
y &= (5-3)^2 - 4 \\
y &= (2)^2 - 4 \\
y &= 4 - 4 \\
y &= 0
\end{align*}
\]

**Example**

\[
y = (x+3)^2
\]

- If we have one root, it is called
Chapter 2 Page 2

Example

\[ y = (x-2)^2 + 2 \]

- no roots
- \( y \neq 0 \)

**Quadratic formula**

The quadratic formula lets you determine the roots/zeros of a quadratic function in general form \( y = ax^2 + bx + c \)

**Formula:** \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

**Example:** Find the roots of

\[ y = x^2 + 4x - 3 \]

**Solution**

\[ 0 = x^2 + 4x - 3 \]

\[ a = 1 \quad b = 4 \quad c = -3 \]

Substitute \( a, b, c \) into the quadratic formula
\[ x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-3)}}{2(1)} \]

\[ x = \frac{-4 \pm \sqrt{16 - (-12)}}{2} \]

\[ x = \frac{-4 \pm \sqrt{28}}{2} \]

\[ x = \frac{-4 \pm 2\sqrt{7}}{2} \]

\[ x = -2 \pm \sqrt{7} \]

\[ -2 + \sqrt{7} \quad -2 - \sqrt{7} \]

The **discriminant**
- the portion of the quadratic formula inside the square root
- the discriminant determines how many roots/zeros the function has

\[ \text{if: } \]
i) If $b - 4ac < 0$, the function has two real roots.

ii) If $b^2 - 4ac = 0$, the function has one real root (double root).

iii) If $b^2 - 4ac < 0$, no real roots.

Examples: How many roots are there:

9) \[ f(x) = x^2 + 6x + 9 \]
   
   \[ a = 1, \quad b = 6, \quad c = 9 \]
   \[ b^2 - 4ac = (6)^2 - 4(1)(9) = 36 - 36 = 0 \]
   
   **One root**

6) \[ f(x) = x^2 - 2x + 2 \]
   
   \[ a = 1, \quad b = -2, \quad c = 2 \]
   \[ b^2 - 4ac = (-2)^2 - 4(1)(2) = 4 - 8 = -4 \]
   
   **No real roots**

C) \[ f(x) = 2x^2 + 8x + 14 \]
   
   \[ a = 2, \quad b = 8, \quad c = 14 \]
   \[ b^2 - 4ac = (8)^2 - 4(2)(14) = 64 - 112 = -48 \]
- No real roots

Practice

#1 Find the values for $p$ which give the equation $x^2 + px + q$, two real roots.

Solution

If we have two real roots, then

$b^2 - 4ac > 0$

- We know $a = 1$, $b = p$, $c = q$

So $p^2 - 4(1)(q) > 0$

$p^2 - 16 > 0$

$p^2 > 16$

2 sets of answers:

$p > 4$ or $p < -4$

$p > 4$ # 26 - 31